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RESONANCE PROPERTIES OF VAPOR BUBBLES*

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Vapor bubbles in a fluid experiencing radial pulsations created by an acoustic field are considered. It is shown that the resonance frequency of large vapor bubbles practically coincides with the eigenfrequency of adiabatic gas bubbles as determined by the Minnaert formula, while in the case of small vapor bubbles, the presence of capillary effects and phase transitions leads to a new resonance frequency that differs from the Minnaert frequency. A simple analytic formula is obtained that relates the resonance frequency of a vapor bubble and its radius; the formula is in good agreement with the results of a numerical solution of the problem. Ranges of dimensions of bubbles, and frequencies of the acoustic field are given, within which different approximations of the relation between resonance frequency and bubble radius hold true. Numerical computations of the resonance frequency based on the radius of a vapor bubble and resonance dimension of a bubble based on field frequency are presented. It is shown that there exists two resonance frequencies and two resonance dimensions of a vapor bubble within some range of dimension of bubbles and acoustic field frequencies. It has also been found /1,2/ that the dynamics of vapor bubbles in an acoustic field reveal the existance of two resonance dimensions of the bubbles. The existence of a new resonance frequency for vapor bubble different from the Minnaert frequency /3/ has also been established /1/.

The resonance properties of homogeneous, equilibrium vapor bubbles has been previously reviewed /4/, though the resonance frequencies of bubbles were not determined entirely correctly. An attempt has been made /5/ to analytically determine the resonance dimensions of vapor bubbles and to physically explain the nature of the second resonance. However, since surface tension was ignored and other inaccuracies were made, an incorrect formula was derived in the latter article which did not describe the actual values of the resonance dimensions of vapor bubbles, for example, in versions previously considered /1,2/.

1. Oscillations of vapor bubbles in an acoustic field. The formulation of the problem of spherically symmetric processes around steam and gas bubbles has been previously set forth /6/. A system of equations that describe oscillations of a nonequilibrium, thermally-inhomogeneous, homobaric vapor bubble in a viscous fluid has been previously presented /7/. Within the framework of a linear representation, analytic solutions have been obtained /1/ for the free and constrained oscillations of bubbles. We will use this solution to study the resonance properties of steam bubbles.

We assume the pressure amplitude of the acoustic field p_A and the frequency ω are small by comparison with the static pressure in the fluid p_∞:

$$p(\infty) = p_\infty + p_A e^{i\omega t}, \quad p_A \ll p_\infty \tag{1.1}$$

In this case, the radius of the bubble may be described by the real part of the expression

$$R = R_0 (1 + \alpha e^{i\omega t}) \tag{1.2}$$

where α is a complex number, $|\alpha| \ll 1$. In a case of vapor bubbles in an incompressible fluid, the previously obtained solution /1/ has the form

$$\alpha = p_A/S, \quad S = \rho_l \omega^2 R_0^2 + 2\sigma/R_0 - 4i\omega\mu_l - 3/Q \tag{1.3}$$

$$Q = \frac{1 - i a_v \{BG(1-\varkappa) + k(1 + EZ^{1/2})\}/(\omega R_0^2)}{\gamma p_0 + i\omega a_v \rho_{v0} \{BG\varkappa - k(1 + EZ^{1/2})\}/3} \tag{1.4}$$

$$\varkappa = c_p T_0/l, \quad Z = i\omega R_0^2/a_v, \quad E = (a_v/a_l)^{1/2}, \quad B = Z^{1/2} \operatorname{cth} Z^{1/2} - 1, \quad a_v = \lambda_v/\rho_{v0} c_p, \quad a_l = \lambda_l/\rho_l c_l$$

$$p_0 = p_\infty + 2\sigma/R_0, \quad T_0 = T_S(p_0), \quad G = 3(\gamma - 1)(1 - \varkappa), \quad k = 3(\gamma - 1)\varkappa^2 \lambda_l/\lambda_v$$

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Here S is a resonance function; Q, compressibility of a bubble; γ, adiabatic index of vapor; ρ, density; l, specific heat of steam formation; μ, σ, λ, coefficients of viscosity, surface tension, and thermoconductivity, respectively; and c_p heat capacity of steam at constant pressure. $T_S(p)$ is the saturation temperature, and the subscripts "l" and "v" refer to the liquid and steam parameters, respectively, while the subscript "0" refers to parameters in the equilibrium state. In the notation of (1.3), it is assumed that steam is described by the simplest equation of state of an ideal gas. By analyzing (1.3) we obtain

$$\lim_{R_0\to 0}|\alpha| = \lim_{R_0\to\infty}|\alpha| = \lim_{\omega\to\infty}|\alpha| = 0,\quad \lim_{\omega\to 0}|\alpha|\frac{p_\infty}{p_A} = \frac{R_0 p_\infty}{2\sigma},\quad \lim_{R_0\to 0}\beta = \lim_{R_0\to\infty}\beta = \lim_{\omega\to 0}\beta = \lim_{\omega\to\infty}\beta = 0 \qquad (1.5)$$

where β is the phase shift between the oscillations of the bubble radius and pressure at infinity. If $\sigma = 0$,

$$\lim_{R_0\to 0}|\alpha|\frac{p_\infty}{p_A} = \frac{p_\infty}{4\omega\mu_l},\quad \lim_{\omega\to 0}|\alpha|\frac{p_\infty}{p_A} = \infty$$

From (1.5), it follows that if $\sigma\neq 0$, there exists at least one bubble dimension at any finite frequency, such that $|\alpha|$ attains its maximal value. This shows incompetence of the results /4/, according to which there are no resonance vapor bubbles in the case of high enough frequencies of the acoustic field.

From an analysis of (1.3), it is clear that when there are no phase transformations,

$$\lim_{R_0\to 0}\beta = \lim_{\omega\to 0}\beta = \pi$$

$$\lim_{\omega\to 0}|\alpha|\frac{p_\infty}{p_A} = \left[3\gamma + (3\gamma - 1)\frac{2\sigma}{R_0 p_\infty}\right]^{-1}$$

Note that in /8/ it has been found that in the case of a gas bubble of constant mass,

$$\lim_{R_0\to 0}\beta = \lim_{\omega\to 0}\beta = 0$$

These relations, as well as the existence of two resonance dimensions of gas bubbles /8/, are valid only at artificially established pressures in bubbles;

$$p_0 = p_\infty + 2\sigma/R_0 = \text{const}$$

But in that case the static pressure in the fluid p_∞ must decrease with decreasing R_0, and become negative in the case of small bubbles (in water if $R_0 \leqslant 1$ μm). In this case, this type of "resonance" is realized with negative pressures in a fluid $p_\infty \approx -2p_0$, which is, for all practical purposes, impossible to achieve.

The expression for the compressibility of a bubble (1.4) may be appreciably simplified if it is taken into account that the estimate

$$0 < G < 1,\quad 0 < \varkappa < 1,\quad k \sim 1,\quad E \gg 1,\quad \omega\rho_{v0}a_v k\,|1 + Ez^{1/2}|/p_0 \ll 1,\quad |B| \ll k\,|1 + EZ^{1/2}| \qquad (1.6)$$

are valid in the case of most substances over a broad range of varying parameters.

The physical meaning of the last inequality is that where there are phase shifts, the internal heat problem is largely unimportant. After simplifications, we obtain

$$Q = [Z + k(1 + EZ^{1/2})]/\gamma p_0 Z$$

Let us consider the case

$$|Z| \gg k,\quad |Z| \gg kE\,|Z^{1/2}|$$

If the estimate (1.6) holds true, the last of the latter inequalities is stronger and when it holds, i.e., when

$$R_0 \gg k a_v (a_l\omega)^{-1/2} \qquad (1.7)$$

the expression for the resonance function has the form

$$S = \rho_l\omega^2 R_0^2 + 2\sigma/R_0 - 4i\omega\mu_l - 3\gamma p_0(1 - kEZ^{-1/2}) \qquad (1.8)$$

The resonance frequency is found by solving the equation

$$\partial\,|S|/\partial\omega = 0 \qquad (1.9)$$

and proof of the condition

$$\partial^2 |S| / \partial\omega^2 > 0 \quad (1.10)$$

Condition (1.7) assumes that the bubbles are coarse enough and that the influence of thermal- and mass-exchange processes on their dynamics is slight; therefore, we will solve equation (1.9) in the form

$$\omega = \omega_0 (1 + \varepsilon), \quad |\varepsilon| \ll 1, \quad \omega_0 = (3\gamma p_0/\rho_l)^{1/2} R_0^{-1} \quad (1.11)$$

where ω_0 is the eigenfrequency of an adiabatic gas bubble subjected to radial oscillations in an ideal fluid /3/. Substituting (1.11) in (1.9) and using the fact that ε is small, along with the capillary and viscous effects, we obtain

$$\varepsilon = -\frac{1}{2} k a_v (2a_l R_0)^{-1/2} (3\gamma p_0/\rho_l)^{-1/4} \quad (1.12)$$

The correction found for the resonance frequency of large vapor bubbles describes the influence of thermal- and mass-exchange processes on bubble dynamics. Naturally, the correction increases with decreasing R_0.

Analogously, if condition (1.7) is satisfied, we may obtain a formula for the resonance dimension of vapor bubbles oscillating at sufficiently low frequencies of an acoustic field. For this purpose, we solve the equation

$$\partial |S| / \partial R_0 = 0 \quad (1.13)$$

and solution is found in the form

$$R_0 = R_M (1 + \delta), \quad |\delta| \ll 1, \quad R_M = \omega^{-1} (3\gamma p_\infty/\rho_l)^{1/2} \quad (1.14)$$

Substituting (1.8) in (1.13) and using the fact that δ is small, we obtain

$$\delta = -\frac{k a_v}{2} \left(\frac{6\gamma p_\infty a_l}{\omega \rho_l} \right)^{-1/4} \quad (1.15)$$

Let us estimate the range of bubble dimensions and frequencies of the acoustic field, such that the resonance frequencies and dimensions of the bubbles are determined by our relations. Note here that the conditions

$$4\omega\mu_l \ll 3\gamma p_\infty, \quad 2\sigma/R_0 \ll 3\gamma p_\infty \quad (1.16)$$

for actual fluids is usually known to be satisfied whenever (1.7) is satisfied. Substituting (1.11) and (1.14) into (1.7), we obtain

$$R_0^{1/2} \gg k a_v (3\gamma p_\infty a_l^2/\rho_l)^{-1/4}, \quad \omega^{1/2} \ll (3\gamma p_\infty a_l/\rho_l)^{1/4}/(k a_v) \quad (1.17)$$

In the case of water, if $p_\infty = 0.1$ MP, these estimates yield

$$R_0 \gtrsim 10^{-2} \text{ m}, \quad f = \omega/2\pi \lesssim 100 \text{ Hz} \quad (1.18)$$

Substituting (1.12) and (1.15) in (1.8) we may determine the amplitudes $|\alpha| p_\infty/p_A$ at the resonance frequency (M_ω) and at the resonance dimension (M_R):

$$M_\omega = (3\gamma)^{-3/4} (p_\infty/\rho_l)^{1/4} (2R_0 a_l)^{1/2}/(k a_v) \quad (1.19)$$
$$M_R = (2a_l p_\infty/(3\gamma\omega\rho_l))^{1/2}/(k a_v)$$

In the other limiting case of sufficiently small bubbles, in which we have the conditions fulfilled

$$|Z| \ll k |1 + EZ^{1/2}|, \quad E|Z^{1/2}| \gg 1 \quad (1.20)$$
$$\rho_l \omega^2 R_0^2 \ll 2\sigma/R_0, \quad 4\omega\mu_l \ll A R_0 \omega^{1/2}$$

the expression for the resonance function has the form

$$S = 2\sigma/R_0 - A R_0 \omega^{1/2} (1 + i), \quad A = l^2 \rho_{v0}^2 (a_l/2)^{1/2}/(\lambda_l T_0) \quad (1.21)$$

The resonace frequency of a vapor bubble has previously /4,5/ been found by solving the equation $\mathrm{Re}(S) = 0$. Such an approach for determining the resonance frequency is incorrect is clear from the fact that in addition to the real part $\mathrm{Re}(S)$, the imaginary part of the resonance function is also a function of the bubble radius and frequency of the acoustic field;

however, this approach does not lead to major errors /4/ only in the region of large bubble radii and low acoustic field frequencies, at which the Minnaert formula holds true. The same inaccuracy in a different article /5/ led to an incorrect formula that related the resonance frequency of a bubble to its radius.

If conditions (1.20) are satisfied, a simple relation may be obtained that relates the resonance frequency of a vapor bubble and its radius. Solving equation (1.9) for (1.21), and then verifying condition (1.10), we obtain /9/

$$\omega = (\sigma/A)^2 R_0^{-4} \tag{1.22}$$

Note that T_0, ρ_{v0}, l, and consequently A as well are functions of the equilibrium pressure in the bubble p_0 which varies with varying R_0 at constant hydrostatic pressure in the fluid p_∞. However, if condition $2\sigma/R_0 \lesssim p_\infty$ holds, the dependence $A\ (R_0)$ is weak and we may assume that $\omega \sim R_0^{-4}$. In this case, by solving equation (1.13) for (1.21), we may determine the dependence of the resonance dimension of the bubble on the acoustic field frequency:

$$R_0^4 = 2\,(\sigma/A)^2\omega^{-1} \tag{1.23}$$

The resulting dependence is not exactly the reverse of (1.22), as it differs by a numerical coefficient.

Substituting (1.22) and (1.23) in (1.21), we may determine resonance values of the oscillation amplitude $|\alpha|\, p_\infty/p_A$:

$$M_\omega = \frac{p_\infty R_0}{\sqrt{2}\,\sigma}, \quad M_R = \frac{p_\infty \omega^{-1/4}}{2\,[A\sigma(\sqrt{2}-1)]^{1/2}} \tag{1.24}$$

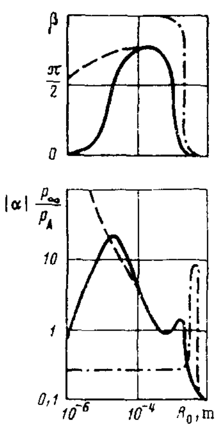

Fig.1

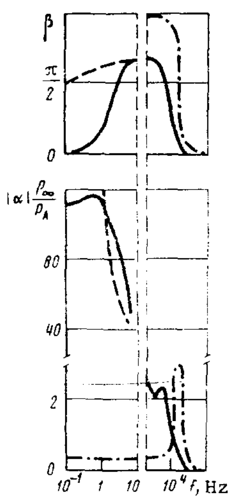

Fig.2

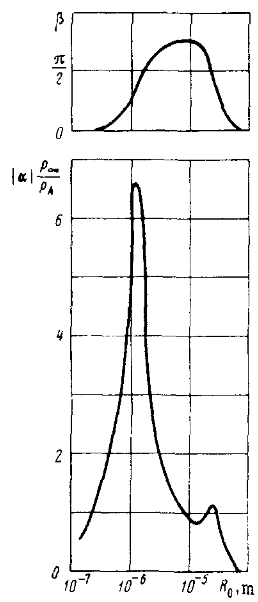

Fig.3

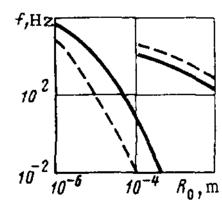

Fig.4

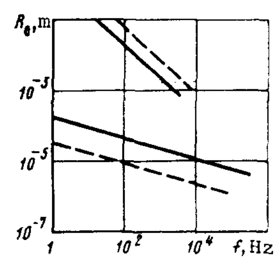

Fig.5

The existance of two vapor bubble resonances is due to frequency variance, since, unlike the case of gas bubbles, the compressibility of small vapor bubbles depends markedly on the oscillation frequency. This also explains why a vapor bubble, unlike a gas bubble, oscillates at low frequencies in phase with fluid pressure far from the bubble.

By estimating the range of bubble dimensions and acoustic field frequencies at which (1.22) and (1.23) hold true, we find that

$$10^{-6}\,\text{m} \lesssim R_0 \lesssim 10^{-4}\,\text{m}, \; 10\ \text{Hz} \lesssim f \lesssim 10^4\ \text{Hz} \tag{1.25}$$

in the case of water at $p_\infty = 0.1$ MP.

The ranges (1.18) and (1.25) overlap. Thus, in the case of water at $p_\infty = 0.1$ MP in the range of acoustic field frequencies 10-100 Hz, two resonance bubble dimensions, as determined by (1.14) and (1.22) obviously exist. In fact, the range of frequencies within which two resonance bubble dimensions exist is much broader.

Fig.1 presents the dependence of the oscillation amplitude $|\alpha| p_\infty/p_A$ and phase β on the bubble radius for the case of bubble oscillations in water at atmospheric pressure. At a frequency $f = 1$ kHz. In Fig.2 may be found the dependence of the bubble oscillation amplitude and phase $R_0 = 0.13$ mm in water at $p_\infty = 0.1$ MP as a function of the acoustic field frequency.

Computations demonstrated that there exists only the Minnaert resonance for large vapor bubbles in water at $p_\infty = 0.1$ MP for $R_0 \geqslant 10^{-3}$ m, while for bubbles with $R_0 \leqslant 10^{-4}$ m, a resonance induced by capillary effects and phase transitions. In the intermediate region, there exist two weak resonances of low quality, while the resonance due to the capillary effect and phase transitions occurs at very low acoustic field frequencies and has a low frequency quality. The broken-line curves in Figs. 1 and 2 correspond to the case $\sigma = 0$. In this case, the range of oscillations of the bubble radius increase without bound as the field frequency or dimension of the vapor bubbles tends to zero, a result that may be interpreted as a resonance. The dot-and-dash line curves correspond to the case of a gas bubble. In this case, there is no second resonance. Computations demonstrated that thought the basic formulas of the present paper are obtained assuming certain constraints on the thermophysical parameters of the system, they remain valid not only for water but also for other fluids, in particular cryogenic fluids over a broad range of pressures and temperatures. Fig.3 presents the dependence of the oscillation, amplitude and phase on the radius of the steam bubble in the case of fluctuations of the bubble radius at a frequency $f = 100$ kHz in liquid nitrogen held at atmospheric pressure. It is clear that two bubble dimensions correspond to a single frequency, and that at low R_0 we have $\beta \simeq 0$.

Note that if surface tension is taken into account and if we require that the liquid-steam bubble system be in thermal and mechanical equilibrium in the absence of any acoustic field (and a fixed hydrostatic pressure p_∞ in the liquid), the temperature of the liquid along the curves in Figs.1 and 3 will be, strictly speaking, variable, since $T_0 = T_S(p_0)$, $p_0 = p_\infty + 2\sigma/R_0$. However, the variation in the temperature in, for example, Fig.3 amounts to only 1.5°K over the interval 10^{-6}, 10^{-4} m.

Fig.4 presents the resonance frequency of a vapor bubble in water and liquid nitrogen (solid and broken-line curves, respectively) as a function of the bubble radius at $p_\infty = 0.1$ MP as computed from (1.3). The two curves have two branches each. The left branches of the curves arise in the region of low R_0 only if both capillary effects and phase transitions are taken into account. In the interval 10^{-5} m $\leqslant R_0 \leqslant 10^{-4}$ m the computed dependences are in good agreement with the solution (1.22). The Minnaert resonance exists in the case $R_0 \geqslant 10^{-4}$ m. The quality of the Minnaert resonance, which decreases with decreasing R_0, is less than unity in the region $R_0 \leqslant 10^{-4}$ m /10/.

In Fig.5 may be found the resonance bubble dimension as a function of field frequency in water and liquid nitrogen at $p_\infty = 0.1$ MP. Over a broad range of frequencies, the function $R_0(f)$ is two-valued and both branches of the curve are described easily by formulas (1.14) and (1.23). The curves confirm the circumstance noted above the existence of a single bubble resonance dimension for every acoustic field frequency.

2. Free oscillation. In the case of low free oscillations of the steam bubble, the bubble radius may be described by the real part of the expression

$$R = R_0(1 + \delta e^{ht}), \quad |\delta| \ll 1$$

The characteristic equation for this case has been previously found /1/.

In an ideal incompressible fluid at states far from the critical at which $\rho_v \ll \rho_l$, the characteristic equation for the bubbles has the form

$$\begin{gathered} H + 3\gamma NH/(H^2 - N\Sigma) + M = 0 \\ H = hR_0^2/a_v, \quad N = p_0R_0^2/(\rho_l a_v)^2, \quad \Sigma = 2\sigma/(R_0p_0) \\ M = k(1 + EH^{1/2}) + 3(\gamma - 1)(1 - \varkappa)^2(H^{1/2}\operatorname{cth} H^{1/2} - 1) \end{gathered} \qquad (2.1)$$

Since it is now clear that there exist two resonance bubble dimensions, and two resonance frequencies as well in some range of bubble dimensions, we may naturally ask about the number of eigenfrequencies of a vapor bubble. Using an independent variable principle, as has been done in /11/, it can be proved that even in the case where there are no phase transitions /12/, and where temperature in the bubble is assumed to be nonhomogeneous, the characteristic equation (2.1) may be simplified and has the form

$$H + 3\gamma NH/(H^2 - N\Sigma) + 3(\gamma - 1)(H^{1/2}\operatorname{cth} H^{1/2} - 1) = 0 \qquad (2.2)$$

as a result of which the latter equation has an infinite number of roots in the left half-plane

($\operatorname{Re} H < 0$). This is because of the periodicity of the cotangent function (which describes the temperature distribution in a bubble /1,12/) in formula (2.2). However, it can be proved that all the roots of equation (2.2), other than the two complex conjugates, are real and much greater in absolute value than the real part of the complex root, a result which thereby proves the validity of results in previous articles /1,12/, in which the characteristic equations were solved numerically, without determining the number and structure of the roots.

In the case of a homogeneous vapor bubble, equation (2.1) may be reduced to the polynomial

$$x^6 + kEx^5 + kx^4 + (3\gamma - \Sigma)Nx^2 - kEN\Sigma x - kN\Sigma = 0 \qquad (2.3)$$
$$x = H^{1/2}$$

From an analysis, it is clear that the latter equation has only a pair of complex conjugate roots in the left complex half-plane H, i.e., a homogeneous vapor bubble has only a single eigenfrequency. Also, in the case of constrained oscillations for some range of dimensions, the range of oscillations of the radius of a homogeneous vapor bubble has a maxima at two acoustic field frequencies.

In the case of very small vapor bubbles ($N \ll 1$), equations (2.1) and (2.3) may sometimes have real negative roots ($H < 0$). This will indicate that a zero eigenfrequency corresponds to the second resonance that appears in oscillations of small bubbles.

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